# Miniprojekt 1: 2D Computer Graphics

David Eriksson, Emil Gustafsson och Linnea Bergman, MT1A, Group 10

March 18, 2016

## 1 Introduction

The aim of this report is to prove how mathematical concepts of linear algebra can be applied in computer graphics. This is done with the use of a two dimensional geometrical interpretation of different transformations that are common tools in computer graphics. The type of transformation covered in this report are shearing, rotation and scaling.

## 2 Method

To illustrate the different transformations an object (*see figure 1*) built up of a number of vectors was created and used. A matrix that described the object was generated from these vectors. The matrix of the object consisted of 21 different vectors.

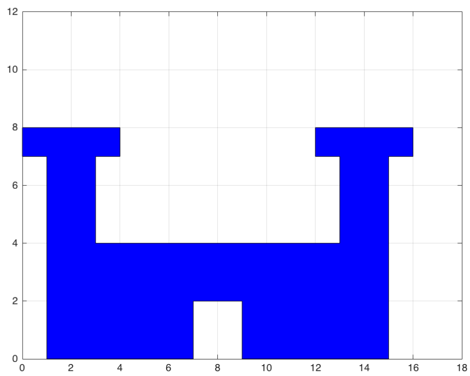

Figure 1: Original figure

To perform any linear transformation, the coordinates of the object was adjusted into *homogeneous coordinates* [1]. The $x, y$ - matrix of the object was expanded with a third row that consisted of ones, otherwise the dimension of the matrices would not agree when multiplied.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \quad (1)$$

To generate a two-dimensional transformation the matrix of the object was multiplied with a given transformation matrix. This technique is known as matrix multiplication. To get the transformed object the order of multiplication of the matrices were important. This is done by multiplying the matrix of the object with the transformation matrix (*see formula 2*).

$$T = M * A \quad (2)$$

Where $T$ represents the transformed object, $M$ the transformation matrix and $A$ the object's matrix.

All transformations were illustrated in two dimensions. The different transformations are described in the following text.

### 2.1 Shearing

Shearing is a way to skew an object. If the object is a square then it is transformed into a parallelogram.

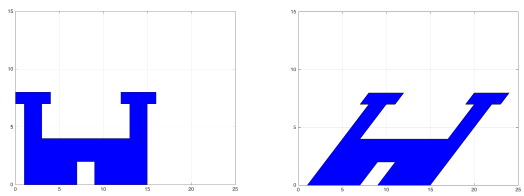

Figure 2: Original and sheared figure

The factor $a$ specifies the degree of skewing that will be applied to the geometric figure.

$$S_x = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \quad (3)$$

$$S_y = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \quad (4)$$

To shear the object along the y-axis instead of the x-axis formula 4 is used.

The vectors that correspond to the parts of the object that remained unchanged after the transformation are the eigenvectors. The eigenvalues of the eigenvectors used in the given equation (*see formula 3 or 4*) were one.

The inverse transformation matrix resulted in a shearing effect in the opposite direction.

## 2.2 Rotation

A rotation is a transformation with a fixed point.

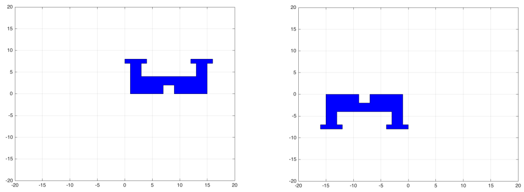

Figure 3: Original and 180° rotated figure

The transformation matrix rotates an object around its origin with an angle $\theta$, which specifies the rotation degree. With a negative $\theta$ the objects rotated clockwise otherwise the rotations were, by default, counterclockwise.

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \tag{5}$$

During rotation there are no vectors mapped to the kernel except the zero vector, therefore the kernel is $\{0\}$. When the transformed object is created the new vectors are part of the image space.

## 2.3 Scaling

Scaling-transformation is when an object is resized either contracted or expanded.

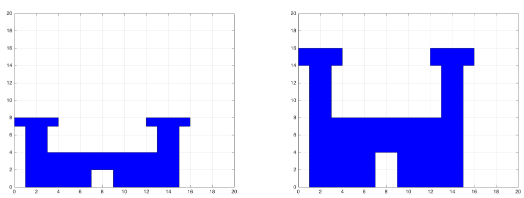

Figure 4: Original and a figure resized, with a factor 2 in height

How much the object was scaled is given by two values, where $a$ scales the object along the y-axis and $b$ the x-axis, whereas the determinant specifies the change in the area of the object.

$$S_c = \begin{pmatrix} a & 0 \\ o & b \end{pmatrix} \tag{6}$$

If any of the given parameters were larger than one ($a > 1$ *or* $b > 1$) then the object expanded. However if negative values are given the object transformes in the 2-4 quadrant of the coordinatesystem. This resulted in a scaled mirrored image of the object. For values between zero and one the object contracted.

## 2.4 Combined transformations

With a combination of transformed matrices several transformation can be done. However, the order in which the object was transformed mattered. With this knowledge it is easier to manipulate a transformation of an object. For example, if we rotate and shear an object the object would first be sheared then rotated.

$$T = R * (S * A) \tag{7}$$

Where $T$ represents the new transformation matrix, $A$ the matrix of the object, $S$ the shearing matrix and $R$ the rotation matrix. In the case given in formula 7 the object is first sheared and then rotated.

# 3 Result and discussion

Linear algebra is useful in 2D computer graphics, especially for transforming and manipulating objects and images. Transformation can manipulate an object by, for example, skewing, rotating or scaling the object. Shearing generates a perspective to the object which gives an illusion of a three-dimensional object. For example, a photographer will often stabilise a photograph with the use of rotation. Lastly, scaling either resizes or generates a mirror effect of the object. All transformation can be combined to create a different effect on the object, which is done with the help of linear algebra and matrix multiplication.

# References

[1] G. Baravdish, *Linjär algebra, TNA002*, Studentlitteratur, Linköping, 2014.